PUMDET-2023

Subject : STATISTICS

(Booklet Number)

Duration : **90** Minutes | No. of Questions : **50** | Full Marks : **100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks **2**. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
3. Use only **Black/Blue ink ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR Sheet**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your signature (as is appeared in Admit Card) in appropriate boxes in the OMR Sheet.
7. The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/signature of the candidate, name of the examination centre. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones, bluetooth devices etc. inside the examination hall. Any candidate found with such prohibited items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question booklet itself. Additional blank pages are given in the question booklet for rough work.
10. Hand over the OMR Sheet to the invigilator before leaving the Examination Hall.
11. Candidates are allowed to take the Question Booklet after examination is over.

Signature of the Candidate : ______________________________
(as in Admit Card)

Signature of the Invigilator : ______________________________

SPACE FOR ROUGH WORK

1. Consider the set $A = [1,3) \cup [2,4)$ and two real numbers a = 2, b = 1.
Which of the following statements is correct ?
(A) b is an interior point of A but not a
(B) a is an interior point of A but not b
(C) Both a and b are interior points of A
(D) None of these two points are interior points of A

2. The series
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2}$$
(A) converges to 2
(B) converges to 1
(C) is divergent
(D) converges to $\sqrt{2}$

3. The value of
$$\int_1^2\int_1^2\int_1^2\int_1^2 \frac{x+y+z-w}{x+y+z+w}\, dx\, dy\, dz\, dw$$
is
(A) 1
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) $\frac{1}{2}$

4. For t > 0, the value of the integral
$$\int_0^{\infty} e^{-tx^2} dx$$
is
(A) $\sqrt{\frac{2\pi}{t}}$
(B) $\sqrt{\frac{\pi}{2t}}$
(C) $\sqrt{\frac{\pi}{t}}$
(D) $\frac{1}{2}\sqrt{\frac{\pi}{t}}$

5. For every positive integer $n \geq 1$, define d_n = number of divisors / factors of n, e.g., for n = 8, there are 4 factors, namely, 1, 2, 4, 8 so $d_8 = 4$. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} d_n x^n$ is
(A) 0
(B) 1
(C) 2
(D) infinity

6. Let U_1, U_2, ………. be i. i. d. Uniform (0, 1) random variables.

Then $\lim_{n\to\infty} P\left(U_1 + U_2 + ... + U_n \le \frac{3}{4}n\right)$

(A) does not exist
(B) exists and equals 1
(C) exists and equals $\frac{1}{2}$
(D) exists and equals 0

7. Suppose $\{X_n : n \ge 1\}$ is a sequence of independent Uniform (0, 1) random variables. Let $Y_n = \min(X_1, ..., X_n)$. Then nY_n converges in distribution to

(A) a N(0, 1) random variable
(B) a N(0, 2) random variable
(C) an exponential random variable with mean 1
(D) an exponential random variable with mean 2

8. Let X be a random variable which is symmetric about zero. Let F be the cumulative distribution function of X. Which of the following statements is always true ?

(A) $F(x) + F(-x) = 1 \ \forall x \in \mathbb{R}$
(B) $F(x) + F(-x) = 0 \ \forall x \in \mathbb{R}$
(C) $F(x) + F(-x) = 1 + P[X = x] \ \forall x \in \mathbb{R}$
(D) $F(x) + F(-x) = 1 - P[X = -x] \ \forall x \in \mathbb{R}$

9. From 10 pairs of shoes in a cupboard, 4 shoes are chosen at random. The probability of selecting at least one of the pairs is

(A) $\frac{97}{323}$
(B) $\frac{95}{323}$
(C) $\frac{224}{323}$
(D) $\frac{99}{323}$

10. For any two events A and B, define $\alpha = P(A \cap B^c)$, $\beta = P(A \cap B)$ and $\gamma = P(A^c)$. Then which of the following relations always holds ?

(A) $\alpha^2 + \beta^2 + \gamma^2 = 1$
(B) $\alpha^2 + \beta^2 + \gamma^2 \le \frac{1}{3}$
(C) $\alpha^2 + \beta^2 + \gamma^2 = \frac{1}{3}$
(D) $\alpha^2 + \beta^2 + \gamma^2 \ge \frac{1}{3}$

11. Suppose X has an exponential density with mean $\theta > 0$. Define Y as follows :
$Y = k$ if $k \le X < k + 1$, $k = 0, 1, 2,$ …….. Then the distribution of Y is

(A) binomial
(B) Poisson
(C) geometric
(D) hypergeometric

12. Consider the quadratic equation $x^2 + 2Ux + V = 0$ where U and V are chosen independently and randomly from the set {1, 2, 3} with equal probabilities. Then the probability that the equation has both roots real, equals

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) $\frac{7}{9}$ (D) $\frac{2}{9}$

13. Suppose (X_1, X_2) follows a bivariate normal distribution with

$E(X_1) = E(X_2) = 0$, $\text{Var}(X_1) = \text{Var}(X_2) = 2$ and $\text{Cov}(X_1, X_2) = -1$. If

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy$, then $P[X_1 - X_2 > 6]$ is equal to

(A) $\Phi(-1)$ (B) $\Phi(-3)$

(C) $\Phi(\sqrt{6})$ (D) $\Phi(-\sqrt{6})$

14. Suppose X_1, X_2 and X_3 are i. i. d. random variables each having a Bernoulli distribution with probability $\frac{1}{2}$. Consider the 2 × 2 matrix $A = \begin{pmatrix} X_1 & 0 \\ X_2 & X_3 \end{pmatrix}$. Then P (A is invertible) equals

(A) 0.20 (B) 0.15

(C) 0.25 (D) 0.75

15. From the 6 letters A, B, C, D, E and F, 3 letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters ?

(A) $\frac{1}{72}$ (B) $\frac{1}{36}$

(C) $\frac{1}{18}$ (D) $\frac{1}{126}$

16. Suppose $X_1, X_2, \ldots, X_n$ are i. i. d. Bernoulli(p) random variables, i.e. $P(X_1 = 1) = p$ and $P(X_1 = 0) = 1 - p$, $0 < p < 1$. Let $Y = \prod_{i=1}^{n} X_i$ and $Z = \max(X_1, X_2, \ldots, X_n)$. Then

(A) $Y \sim$ Bernoulli (p^n) and $Z \sim$ Bernoulli $((1-p)^n)$

(B) $Y \sim$ Bernoulli $((1-p)^n)$ and $Z \sim$ Bernoulli $((1-p)^n)$

(C) $Y \sim$ Bernoulli $((1-p)^n)$ and $Z \sim$ Bernoulli (p^n)

(D) $Y \sim$ Bernoulli (p^n) and $Z \sim$ Bernoulli $(1-(1-p)^n)$

17. Let the distribution function F of a random variable X be given by

$$F(x)=\begin{cases}0 & \text{if } x<0\\ \dfrac{x+1}{2} & \text{if } 0\le x<1\\ 1 & \text{if } 1\le x\end{cases}$$

Then $P\left(-3<X\le\frac{1}{2}\right)$ is

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$

(C) 0 (D) $\frac{1}{6}$

18. Let $B=\begin{pmatrix}2 & -1 & 3 & 4 & 5\\ 0 & 1 & -1 & 2 & 3\end{pmatrix}$

Then rank $\left(B^TB\right)$ is

(A) 2 (B) 5

(C) 1 (D) 3

19. Let $A=\left(\left(a_{ij}\right)\right)$ be an $n\times n$ real matrix. Then which of the following is NOT true?

(A) If A is invertible, $tr\left(A^TA\right)\ne 0$

(B) If $tr\left(A^TA\right)\ne 0$, then A is invertible

(C) If $tr\left(A^TA\right)<n^2$, then $\left|a_{ij}\right|<1$ for some i, j

(D) If $tr\left(A^TA\right)=0$, then A is the zero matrix

20. The inverse of the matrix

$$A=\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & 0\\ \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{-3}{\sqrt{12}}\end{pmatrix}$$

(A) does not exist (B) is A^T

(C) is $(A^T)^2$ (D) is A^2

21. Let X_1, X_2 and X_3 be independently and identically distributed Bernoulli(p) random variables. Which of the following is NOT a sufficient statistic for p ?

(A) $(X_1, X_1 + X_2, X_1 + X_2 + 2X_3)$ (B) $(X_1, X_1 + X_2, X_3)$

(C) $(X_1 + X_3, X_1 + X_2 + 2X_3)$ (D) $X_1 + 17X_2 + X_3$

22. Let X_1, X_2, ..., X_n be a random sample from Uniform $(\theta, \theta + 1)$. Then which of the following is NOT an MLE of θ ?

(A) $\dfrac{X_{(1)} + X_{(n)} - 1}{2}$ (B) $\dfrac{2X_{(1)} + 3X_{(n)} - 3}{5}$

(C) $\dfrac{X_{(1)} + X_{(n)} + 1}{2}$ (D) $\dfrac{3X_{(1)} + 2X_{(n)} - 2}{5}$

23. Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated random variables with common unknown variance σ^2 and expectations given by

$$E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2),$$
$$E(Y_3) = \beta_1 - \beta_2 = E(Y_4),$$

where β_1, β_2 and β_3 are unknown parameters. Define

$e_1 = \dfrac{1}{\sqrt{2}}(Y_1 - Y_2)$ and $e_2 = \dfrac{1}{\sqrt{2}}(Y_3 - Y_4)$. An unbiased estimator of σ^2 is

(A) $\dfrac{1}{2}(e_1{}^2 + e_2{}^2)$ (B) $\dfrac{1}{2}(e_1{}^2 - e_2{}^2)$

(C) $\dfrac{1}{4}(e_1{}^2 + e_2{}^2)$ (D) $\dfrac{1}{4}(e_1{}^2 - e_2{}^2)$

24. Let X and Y follow χ^2-distributions with the same degrees of freedom, independently. Then $E\left(\dfrac{X}{X+Y}\right)$ is

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$

(C) $\dfrac{3}{4}$ (D) 1

25. Let X_1, X_2, ..., X_{10} be a random sample from Normal (0, 1). Let N be the number of X_i's less than or equal to 0. Then Var(N) is

(A) 10 (B) 5

(C) 2.5 (D) 1.25

26. Let X be a random variable such that $E(X)=E(X^2)=1$. Then $E(X^3)$ is

(A) 0 (B) 1
(C) 2 (D) 3

27. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, 1)$. Let $\overline{X}=(1/n)\sum_{i=1}^{n}X_i$. Then $\mathrm{Var}\left[E\left(X_1+X_2 \mid \overline{X}\right)\right]$ is

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$
(C) $\frac{3}{n}$ (D) $\frac{4}{n}$

28. Let $X_1, X_2, \ldots, X_{10} \overset{iid}{\sim}$ Cauchy (0, 1). Let R_i be the rank of X_i for i = 1, 2, 3, …., 10. Then

(A) $E(R_{10} - R_1) = 0$ (B) $E(R_{10} - R_1) = 9$
(C) $E(R_{10} - R_1) = \frac{10}{\pi}$ (D) $E(R_{10} - R_1)$ does not exist

29. Consider the problem of testing
$H_0 : X \sim$ Poisson (2) vs. $H_1 : X \sim$ Exp (2).
We reject H_0 against H_1 based on a single observation x if and only if x is not a non-negative integer. Then

(A) P(Type -I error) = 0.5 and P(Type -II error) = 0.5
(B) P(Type -I error) = 0 and P(Type -II error) = 0
(C) P(Type -I error) = 0 and P(Type -II error) = 0.5
(D) P(Type -I error) = 1 and P(Type -II error) = 0.5

30. Let $x_1 = 3, x_2 = 4, x_3 = 3.5, x_4 = 2.5$ be the observed values of a random sample from the probability density function

$f(x \mid \theta)=\frac{1}{2}\left[\frac{1}{\theta}e^{-\frac{x}{\theta}}+e^{-x}\right], x > 0, \theta > 0.$ Then the method of moment estimate of θ is

(A) 3.5 (B) 4
(C) 5.5 (D) 6.5

31. Suppose $X_1, X_2, ., X_n$ are independent random variables and $X_k \sim N(0, k\sigma^2)$, k = 1, 2, …, n where σ^2 is unknown. The maximum likelihood estimator for σ^2 is

(A) $\frac{1}{n}\sum_{k=1}^{n}{X_k}^2$ (B) $\frac{1}{n}\sum_{k=1}^{n}\left(X_k-\overline{X_n}\right)^2$
(C) $\frac{1}{n}\sum_{k=1}^{n}\frac{{X_k}^2}{k}$ (D) $\sum_{k=1}^{n}{X_k}^2 \Big/ \sum_{k=1}^{n}k$

32. Suppose $X = (X_1, X_2, X_3)^T$ has $N_3(\mu, \Sigma)$ distribution where $\mu = (-3, 1, 4)^T$ and $\Sigma = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$. Then which of the following pairs of random variables are NOT independent ?

(A) X_2 and X_3

(B) $\frac{X_1 + X_2}{2}$ and X_3

(C) X_2 and $X_2 - \frac{5}{2}X_1 - X_3$

(D) X_1 and $\frac{X_2 + X_3}{2}$

33. Let the random variable 'X' have binomial distribution with parameters 3 and θ. A test of hypothesis $H_0 : \theta = \frac{3}{4}$ against $H_1 : \theta = \frac{1}{4}$ rejects H_0 iff $X \leq 1$. Then the test has

(A) size $= \frac{5}{32}$, power $= \frac{27}{32}$

(B) size $= \frac{5}{32}$, power $= \frac{18}{32}$

(C) size $= \frac{15}{32}$, power $= \frac{27}{32}$

(D) size $= \frac{1}{32}$, power $= \frac{31}{32}$

34. The name of the header file to which puts () belongs is

(A) math.h

(B) stdio.h

(C) iomanip.h

(D) stdlib.h

35. Let X_1 and X_2 be independently and identically distributed as normal with mean θ and variance unity. Then $E(\bar{X} \mid X_1 = 1)$ will be

(A) $\frac{\theta+1}{2}$

(B) $\frac{\theta-1}{2}$

(C) $\frac{\theta}{2}$

(D) $\frac{\theta}{\sqrt{2}}$

36. The diagnostic rating scale based on a mammogram to detect breast cancer as definitely normal, probably normal equivocal, probably abnormal, definitely abnormal uses

(A) Ratio scale

(B) Interval scale

(C) Nominal scale

(D) Ordinal scale

37. For a given bivariate data set (x_i, y_i), i = 1, 2, …, n; the squared correlation coefficient (r^2) between x^2 and y is found to be 1. Which of the following statement is most appropriate ?

(A) In the (x, y) scatter diagram, all points lie on a straight line.

(B) In the (x, y) scatter diagram, all points lie on a curve $y = x^2$.

(C) In the (x, y) scatter diagram, all points lie on a curve $y = a + bx^2$, $a > 0$, $b > 0$.

(D) In the (x, y) scatter diagram, all points lie on a curve $y = a + bx^2$, a and b are any real numbers.

38. Suppose your data produces the regression of y on x as y = 10 + 3x. Scale y by multiplying by 0.9 and do not scale x. The new intercept and slope estimates will be respectively

(A) 10, 3 (B) 9, 3
(C) 9, 2.7 (D) 11.11, 3.33

39. The data on annual income is summarised in the following frequency distribution table :

Annual Income (in '000 ₹)	80 - 100	100-120	120-140	140-160	160-180
No. of families	5	10	15	10	5

The value of m_3 (third order central moment) comes out as

(A) 2.25 (B) 0
(C) – 2.25 (D) 1

40. For a set of 8 observations if 3 and 23 are the minimum and maximum values respectively, then the minimum and maximum possible values of the variance are respectively

(A) 25 and 100 (B) 50 and 150
(C) 20 and 80 (D) 50 and 80

41. Suppose a finite population consists of N units numbered 1, 2, ….., N. Two units are drawn using SRSWOR and let (X, Y) be the numbers of the sampled units. If Z = min (X, Y), then E(Z) is

(A) $\frac{N-1}{6}$ (B) $\frac{N}{6}$
(C) $\frac{N(N+1)}{6}$ (D) $\frac{N+1}{6}$

42. Consider the following model

$(y_1, y_2, y_3, y_4)^T = (1, 1, 1, 0)^T \beta_1 + (1, 1, 0, 1)^T \beta_2 + (e_1, e_2, e_3, e_4)^T,$

$(e_1, e_2, e_3, e_4)^T \sim N_4(0, \sigma^2 I_4)$

Then among the following, which is most preferable for deriving a confidence interval for $\beta_1 - \beta_2$?

(A) $y_3 - y_4$ (B) $y_3 - y_4 + y_1 - y_2$
(C) $y_1 - 2y_2 + y_3$ (D) $y_1 - y_2$

43. In a 2^2 factorial experiment the main effect A is given by

(A) $\frac{1}{2}\{(ab)+(a)-(b)-(1)\}$ (B) $\frac{1}{2}\{(ab)-(a)+(b)-(1)\}$
(C) $\frac{1}{2}\{(ab)+(a)+(b)+(1)\}$ (D) $\frac{1}{2}\{(ab)-(a)-(b)+(1)\}$

44. If the degrees of freedom for the error sum of squares in a Latin square design is 6, then the order of the design is

(A) 3×3 (B) 4×4

(C) 5×5 (D) 6×6

45. In a randomized block design with 4 blocks and 5 treatments having one missing value, the error degrees of freedom will be

(A) 12 (B) 11

(C) 10 (D) 9

46. A simple random sample of size 3 is drawn with replacement from a population of size N (> 7). The probability that the sample contains exactly 2 distinct units is

(A) $1/N^2$ (B) $(N - 1)(N - 2)/N^2$

(C) $3(N - 1)/N^2$ (D) $(N - 1)/N$

47. Which of the following is NOT a division of the Central Statistics Office (CSO) ?

(A) National Accounts Division (B) Demography Division

(C) Social Statistics Division (D) Economic Statistics Division

48. Given that the Total Fertility Rate (TFR) is 2620 per thousand and sex-ratio at birth is 1 male to 1 female, then the Gross Reproduction Rate (GRR) will be

(A) 1.000 (B) 1.310

(C) 2.620 (D) 5.240

49. How many moving average values are feasible when 3-year moving averages are calculated from the values for the years 2010 to 2022 ?

(A) 10 (B) 11

(C) 12 (D) 13

50. If μ and σ are the process mean and standard deviation respectively, then the control limits $\mu \pm 3\sigma$ are known as

(A) modified control limits (B) natural control limits

(C) specified control limits (D) artificial control limits

SPACE FOR ROUGH WORK